Tensile and compressive moduli of fibres using a two-component beam system

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Serious difficulties are associated with the measurement of the compressive modulus of fibres owing to their tendency to buckle during loading. A novel technique is described, based on the theory of two-component materials for determining both the tensile and compressive moduli of reinforcing fibres in composites. It was found, particularly with Kevlar 49, that the two moduli are significantly different.

1. Introduction

For many composite engineering applications, information regarding both tensile modulus and compressional modulus of reinforcing fibres is often required. In the case of fibres, the former can be measured relatively easily; however, because of the very small diameter it is extremely difficult to measure the axial compressive modulus without buckling. As a result many workers [1–3] have assumed that the tensile and compressive moduli are identical. However, it is the purpose of this paper to report on a novel technique designed to measure both parameters.

When a material is subjected to bending, one section is in tension and the other part away from the neutral axis is in compression. It was decided, therefore, to fabricate an asymmetric composite consisting of fibres embedded along one side of a rod. During bending of the composite, the fibres could be subjected to either tension or compression depending on whether they were above or below the neutral axis.

The asymmetric composite rod (with the fibres uppermost) was fixed at one end and the free end progressively loaded. From measurements of the deflection of the free end of the cantilever, using the theory of two-component materials [4], the tensile modulus of the fibre may be determined. The rod was then inverted and the procedure repeated. In this way the compressive modulus of the fibres could also be found.

2. General theory

Consider a composite rod of two materials 1 and 2 (Fig 1a). Assuming that (a) no slippage takes place between the two materials and (b) the cross-section remains planar during bending, the theory of solid beams will apply. Thus the strain on the longitudinally arranged fibres will be proportional to their distance from the neutral axis. Moreover, for any bending curvature, $1/\rho$, within the elastic range of the material, the normal stress in material 1, at a distance y from the neutral axis is given by

$$\rho = \frac{E_1 y}{\rho} \quad (1)$$

If an element has a cross-sectional area of $\mathrm{d}A = b\mathrm{d}y$ then the force acting is given by

$$\mathrm{d}f_1 = \frac{E_1}{\rho} b y \mathrm{d}y \quad (2)$$

where $\mathrm{d}y$ is the thickness of the element and E_1 is the modulus of elasticity of material 1. Similarly, for an elemental area of material 2, the force acting is given by

$$\mathrm{d}f_2 = \frac{E_2}{\rho} b y \mathrm{d}y \quad (3)$$

where E_2 is the modulus of elasticity of material 2. In pure bending, these elemental forces ($\mathrm{d}f_1$ and $\mathrm{d}f_2$) summed over the total areas of materials 1 and 2, respectively, must have a net resultant force equal to zero, and the sum of their moments about the neutral axis must be equal to the moment of resistance developed by the section. Without actually making these summations, the results will be unchanged if Equation 2 is written in the equivalent form

$$\mathrm{d}f_1 = \frac{E_2}{\rho}\left(\frac{E_1}{E_2} b\right) y \mathrm{d}y \quad (4)$$

Equation 4 shows that we may regard the portion of material 1, of width b, as equivalent to a beam of material 2 with reduced width $d = (E_1/E_2)b$ for the case $(E_1 < E_2)$ as shown in Fig. 1b. Under a given load, the composite section in Fig. 1a and the transformed section in Fig. 1b will have the same moment of resistance. Hence using the transformed section, the problem of bending a beam of two materials is reduced to the bending of a T-section beam of material 1 only.

In order to calculate the moduli of one component, it is necessary to know the moduli of the second component. Spring steel was chosen as the second

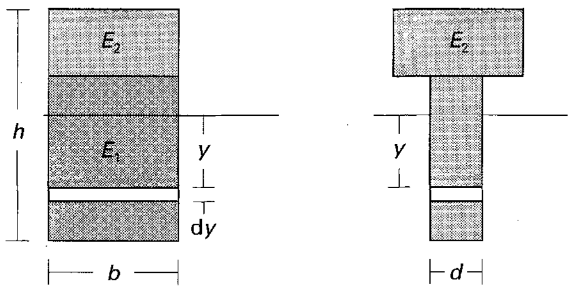

Figure 1 (a) Real composite section. (b) Transformed composite section.

component primarily because it behaves elastically and also its tensile and compressive moduli are considered to be identical [5, 6].

3. The modulus of spring steel

The modulus was determined by loading a simple cantilever consisting of a spring steel strip fixed at one end and measuring the deflection, δ, at the loading point. The plot of load versus deflection was a straight line through the origin. From the slope of the line the rigidity, R, of the spring steel strip was calculated and then the elastic modulus, E_s, was determined using the classical equation

$$R = E_s I = \frac{PL^3}{3\delta} \quad (5)$$

where P is the load, L is the length of the steel strip (99.3 mm) and I is the moment of inertia which is given by

$$I = \frac{bh^3}{12} \quad (6)$$

where b is the width of the steel strip (12.9 mm) and h is the depth of the steel strip (0.65 mm). A value of 181.4 GPa was calculated for the modulus of the spring steel with a possible error of $\pm 6\%$.

4. The moduli of perspex

Having determined the modulus of spring steel it is now possible to determine the two moduli for the other component of an asymmetric composite. In this case perspex was chosen because loading tests over the required range showed it to be elastic with little or no creep.

4.1. Theory

Consider a rectangular cross-section composite rod of spring steel and perspex.

4.1.1. Case A, perspex uppermost

Fig. 2 shows (a) the cross-section of the real composite rod and (b) The transformed section, where a is the depth of the spring steel, b the width of the rod, h the depth of the perspex, E_s the modulus of the spring steel, E_{pt} the modulus of the perspex in tension, $k = E_s/E_{pt}$.

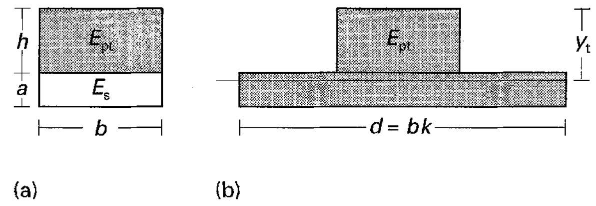

Figure 2 Perspex in tension. (a) Real section, (b) transformed section.

Generally the distance of the neutral axis from a given parallel axis is given by

$$Y = \frac{\sum_{i=1}^{\infty} y_i A_i}{\sum_{i=1}^{\infty} A_i} \quad (7)$$

where A_i is the area of the element, i, and y_i is the distance of the centroid of the element i from the measuring axis.

The distance between the neutral axis and top of the perspex, y_t (Fig. 2b) is found to be

$$y_t = \frac{h^2/2 + ak(h + a/2)}{h + ak} \quad (8)$$

The moment of inertia about the neutral axis of the transformed section (Fig. 2b) is given by [4]

$$I_t = \frac{bh^3}{3} + \frac{da^3}{3} - [(bh + da)(y_t - h)^2] \quad (9)$$

Because the aim of this experiment is to calculate the moduli of the perspex in tension and in compression, the entire perspex region of the composite must be on one side of the neutral axis (i.e. the neutral axis must not intersect the perspex region).

Using the composite rod as a simple cantilever of length, L, and measuring the deflection at the end of the rod, δ, under load, P, allows calculation of rigidity, R_t. In the case of perspex uppermost (perspex in tension)

$$R_t = E_{pt} I_t = \frac{PL^3}{3\delta} \quad (10)$$

where E_{pt} is the tensile modulus of perspex.

In Equations 8 and 9, the values of y_t and I_t are a function of $k = E_{pt}/E_s$. A numerical solution using a computer program allows calculation of E_{pt}. The program assigns an initial value for k and then increments it. At each increment, the value of Q is calculated

$$Q = R_t - E_{pt} I_t \quad (11)$$

when Q is zero the value of E_{pt} can be determined.

4.1.2. Case B, perspex innermost

In this case the perspex is under compression. Fig. 3 shows (a) the real cross-section of the rod and (b) the

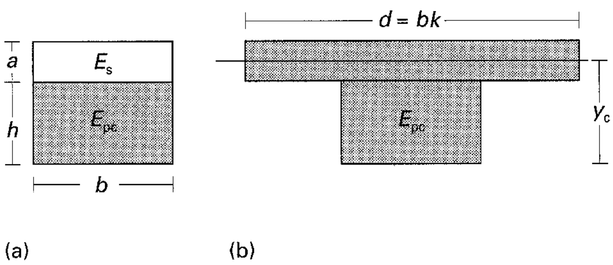

Figure 3 Perspex in compression. (a) Real section, (b) transformed section.

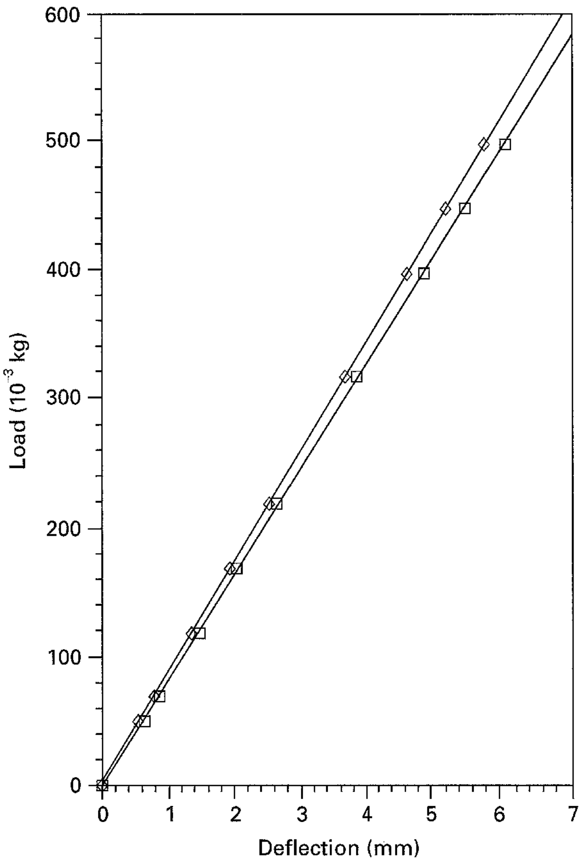

Figure 4 The perspex–steel composite test. (□) Tension, (◇) compression.

transformed section where E_{pc} is the modulus of the perspex in compression and y_c is the distance between the neutral axis and the bottom of the perspex (Fig. 3b). Similar experimental procedures and numerical solutions as described in case A yield the modulus of perspex in compression E_{pc}.

4.2. Experimental procedure

Strips of perspex and spring steel having rectangular cross-sections of 12.9 mm × 3.0 mm and 12.9 mm × 0.65 mm respectively, were bonded together using acrylic resin to ensure that no slippage takes place during bending. After curing the resin, the composite rod was held in a vice and the length of the cantilever (119.1 mm) measured using a travelling microscope. Cantilever loading–deflection experiments were then carried out for two cases, i.e. perspex uppermost (perspex in tension) and perspex innermost (perspex in compression).

4.3. Results

Fig. 4 shows the plots of load, P, against deflection, δ, for both cases. The data points fall on straight lines through the origin. Moreover it is clear that the slopes for the two cases are slightly different, indicating that perspex has indeed slightly different moduli in tension and compression. From the two rigidities and using the theory described previously, the values of 2.79 and 2.93 GPa were calculated for the moduli of the perspex in tension, E_{pt}, and compression, E_{pc}, respectively.

5. Tensile and compressive moduli of the fibres

5.1. Theory

Consider now a composite in the form of a perspex rod containing a slot filled with fibres as shown in Fig. 5a. E_{pt} and E_{pc} refer to the tensile and compressive moduli of the perspex, respectively. E_{ft} is the modulus of the fibre in tension, E_{fc} is the modulus of the fibres in compression and f is the depth of the fibres in the slot. Providing that the fibres remain the same vertical distance from the neutral axis, then theoretically the moment of inertia of the composite will remain constant even if the fibres move laterally in the slot. Thus the cross-section of the fibre region can be considered as a solid rectangle of height f and width z (Fig. 5b) where

$$z = A/f \tag{12}$$

and A = total cross-sectional area of the fibres.

The remaining very small area of the slot contains the acrylic bonding resin which is assumed to have the same modulus as perspex.

5.1.1. Case A, slot uppermost containing fibres

During bending, the uppermost region of the composite is under tension. Consider Fig. 5a (cross-section of a two component rod), where b is the width of the perspex rod, h is the depth of the slot, w is the width of the slot, f is the depth of the fibres, and a is the depth of the perspex rod.

$$m = h - f \tag{13}$$

$$c = (b - w)E_{pt}/E_{ft} \tag{14}$$

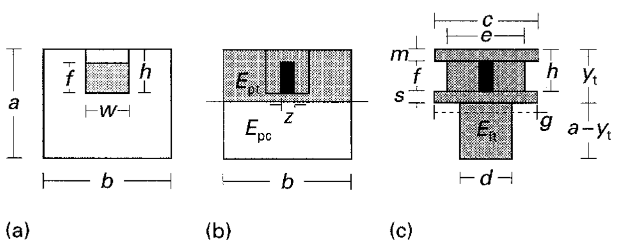

Figure 5 Fibres in tension. (a) Real section, (b) intermediate transformed section, (c) final transformed section with respect to E_{ft}.

$$e = z + (b - z)E_{pt}/E_{ft} \quad (15)$$

$$g = bE_{pt}/E_{ft} \quad (16)$$

(when the slot is completely filled with resin then $c = g$)

$$d = bE_{pc}/E_{ft} \quad (17)$$

The distance of the neutral axis from the top of the composite is given by

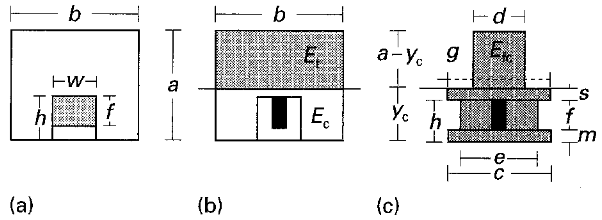

Figure 6 Fibres in compression. (a) Real section, (b) intermediate transformed section, (c) final transformed section with respect to E_{fc}.

$$y_t = \frac{mc(m/2) + ef[m + (f/2)] + g(y_t - h)[h + (y_t - h)/2] + d(a - y_t)[y_t + (a - y_t)/2]}{mc + ef + g(y_t - h) + d(a - y_t)} \quad (18)$$

Simplifying gives

$$(g - d)y_t^2 + 2(mc + ef - gh + da)y_t - (cm^2 + 2efm + ef^2 - gh^2 + da^2) = 0 \quad (19)$$

From the above quadratic equation, y_t can be determined.

The moment of inertia of the transformed section (Fig. 5c) is

$$I_{ft} = \frac{cm^3}{12} + \frac{ef^3}{12} + \frac{gs^3}{3} + \frac{d(a - y_t)^3}{3} + \mathrm{cm}(y_t - m/2)^2 + ef(s + f/2)^2 \quad (20)$$

where $s = y_t - h$

As the aim of this experiment is to determine the moduli of the fibres in compression and tension, the entire fibre region of the composite must be on one side of the neutral axis (i.e. the neutral axis must not intersect the fibre region).

The composite rod was fixed at one end and the other end was loaded and the rigidity, R_t, calculated as shown previously.

As can be seen from Equation 20, I_{ft} is a function of E_{ft}. A numerical solution using a computer program allows calculation of E_{ft}. The program assigns a provisional E_{ft} value and then increments it. At each increment, Q is calculated

$$Q = R_t - E_{ft}I_{ft} \quad (21)$$

where R_t is the rigidity from the cantilever experiment, I_{ft} the moment of inertia from Equations 19 and 20, and E_{ft} the tensile modulus of the fibre. When Q is zero the real value of E_{ft} can be determined.

5.1.2. *Case B, slot innermost containing fibres*

In this case, bending of the composite rod causes the fibres to be compressed. E_{fc} refers to the modulus of the fibre in compression, and I_{fc} is the moment of inertia of the transformed section and y_c is the distance of the neutral axis from the top of the slot (Fig. 6a, b, and c).

The final transformed section in this case is shown in Fig. 6c. It can be shown that in this case

$$m = h - f \quad (22)$$

$$c = (b - w)E_{pc}/E_{fc} \quad (23)$$

$$e = z + (b - z)E_{pc}/E_{fc} \quad (24)$$

$$g = bE_{pc}/E_{fc} \quad (25)$$

when the slot is completely filled with resin then $c = g$

$$d = bE_{pt}/E_{fc} \quad (26)$$

$$y_c = \frac{mc(m/2) + ef[m + (f/2)] + g(y_c - h)[h + (y_c - h)/2] + d(a - y_c)[y_c + (a - y_c)/2]}{mc + ef + g(y_c - h) + d(a - y_c)} \quad (27)$$

where y_c is the distance of the neutral axis of the transformed section (Fig. 6c) from the top of the slot. Simplifying Equation 27 gives

$$(g - d)y_c^2 + 2(mc + ef - gh + da)y_c - (cm^2 + 2efm + ef^2 - gh^2 + da^2) = 0 \quad (28)$$

From the above quadratic equation y_c can be determined.

The moment of inertia I_{fc}, of the transformed section (Fig. 6c) is found to be

$$I_{fc} = \frac{cm^3}{12} + \frac{ef^3}{12} + \frac{gs^3}{3} + \frac{d(a - y_c)^3}{3} + cm(y_c - m/2)^2 + ef(s + f/2)^2 \quad (29)$$

where $s = y_c - h$.

Similar numerical calculations and similar experimental procedures as described in Case A yields the modulus of the fibre in compression, E_{fc}.

5.2. Experimental procedure

Experiments were carried out on three types of aramid fibres, Kevlar 49 (spool 1), Kevlar 29 and Technora. Individual perspex rods (~ 200 mm × 6 mm × 8 mm) were milled along one face to produce a slot of rectangular cross-section (~ 200 mm × 2 mm × 2 mm). The slot in the perspex was then filled with fibre as follows. A tow of fibres was placed linearly in a paper tray and covered with a two component acrylic resin using an adhesive gun fitted with a mixing nozzle. After thorough impregnation, the tow was transferred to the slot in the perspex rod and held taut. Further plies of the same tow were placed in the slot in the same way. The adhesive set in about 10 min and the assembly (perspex + fibres) left for 3 days at room temperature to harden fully. Composite rods were produced for each fibre type. The dimension of the rods together with the fibre depths (see Fig. 6c) were measured using a travelling microscope.

One end of a composite rod was fixed firmly in a vice to form a simple cantilever. The cantilever was then progressively loaded and at each stage the deflection of the rod at the loading point was measured. The cantilever tests were carried out for the two cases, i.e. fibre uppermost (tension) and fibre innermost (compression).

The total fibre cross-sectional areas were also required. These were calculated by weighing known lengths of tow and using the equation

$$A = \frac{nw}{l\rho} \qquad (30)$$

where A is the total fibre cross-sectional area, l is the length of the tow, n is the number of tows, w is the weight of one tow, and ρ is the density (1450 kg m^{-3}).

5.3. Results

The plots of load, P versus deflection, δ, for Kevlar 49 (spool 1), Kevlar 29 and Technora are shown in Figs. 7, 8 and 9, respecively. In the case of fibres uppermost in the bent composite rods, all will be under tension and, as can be seen, the data points form straight lines through the origin with very high coefficients of regression.

In the case of Kevlar 49 fibres innermost in the bent rod, the data points are initially in a straight line, but then tend to deviate. This effect is to be expected when the fibres under compression undergo a transition from elastic to plastic deformation. The transition point in compression corresponds to a strain of approximately 0.2%. In the case of Kevlar 29 and Technora the applied loads did not permit attainment of the critical strain.

The slopes of the straight regions of the graphs give the values of rigidity (R_t and R_c) for both cases. From the rigidity values and using the theory described previously, the average values of the moduli of the fibres in tension, E_{ft}, and in compression E_{fc}, for three types of fibres were calculated. The results are shown in Table I.

Using the computer program, the values of y_t and y_c were calculated for each experiment. It was established that in all cases the fibres were either completely

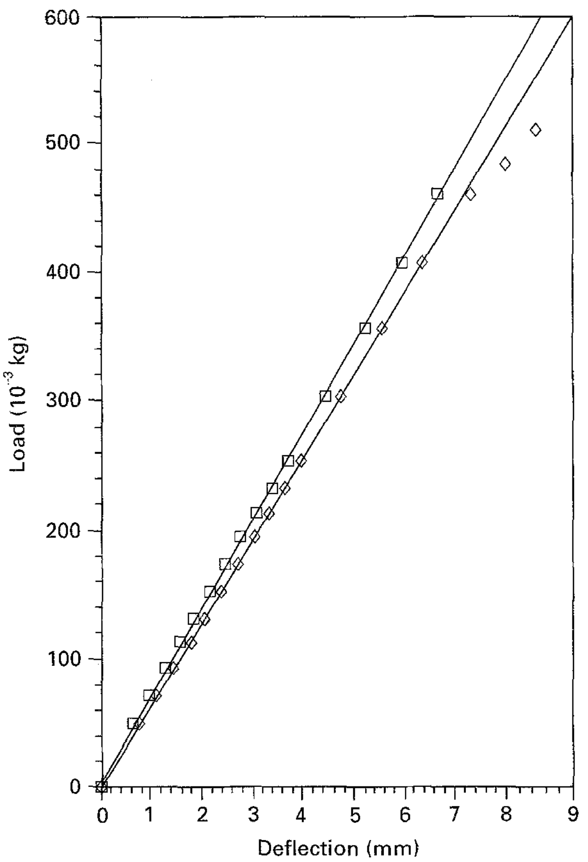

Figure 7 The Kevlar 49 (spool 1)–Perspex composite test. (□) Tension, (◇) compression.

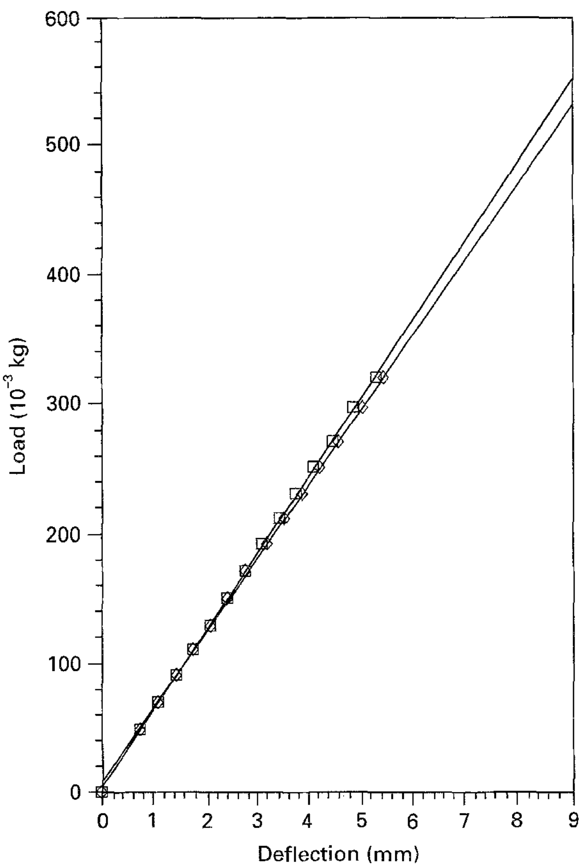

Figure 8 The Kevlar 29–Perspex composite test. (□) Tension, (◇) compression.

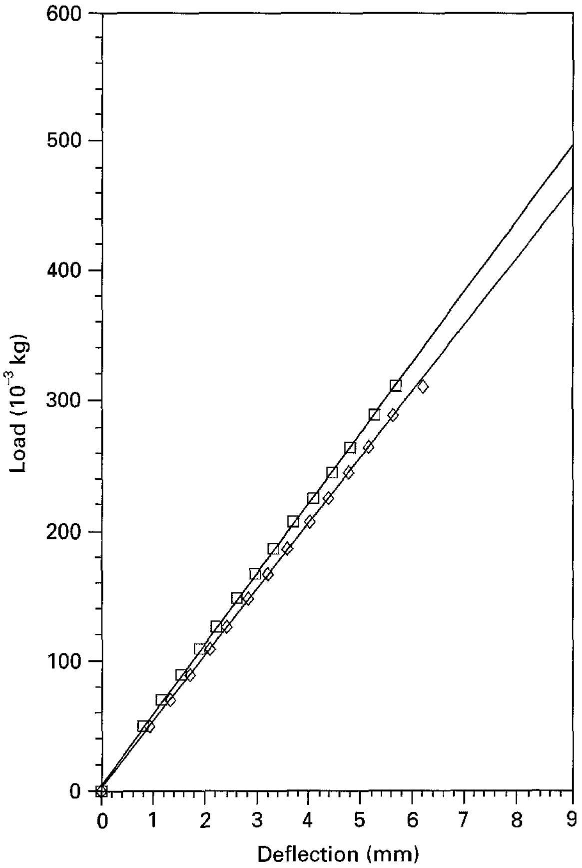

Figure 9 The Technora–Perspex composite test. (□) Tension, (◇) compression.

above (tension) or completely below (compression) the neutral axis.

6. Conclusion

The perspex deforms elastically within the loading conditions (it returns to its initial form after unloading) and the compressive modulus of the perspex is slightly higher than the tensile modulus.

The results of the bending of perspex + fibre assemblies show that the aramid fibres generally have a lower modulus in compression than in tension. Recently other workers [7] using a micro-composite method to measure the stress–strain property of the fibres in longitudinal compression have reached a similar conclusion. The difference between the tensile and compressive moduli is much higher in the case of Kevlar 49 than in the Kevlar 29 and Technora.

The calculated values of tensile moduli from the composite experiments are slightly lower than the tensile moduli obtained in the Instron tests of the single fibres. One possible reason for this may be due to the large number of fibres in a tow, some of which will not be perfectly aligned within the composite. In this case there would be unequal load sharing leading to a lower value for the modulus.

TABLE I Mechanical data

	Instron tensile modulus (GPa)	Average E_{ft} (GPa)	Average E_{fc} (GPa)	E_{fc}/E_{ft} (%)
Kevlar 49	133	118.2	85.0	71.9
Kevlar 29	77	68.8	64.7	94.4
Technora	102	98.7	89.9	90.2

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